

# Introduction to Student Modeling and Bayesian Knowledge Tracing

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Universitat Ramon Llull



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- Student modeling and its context
- Bayesian Knowledge Tracing
- Where is Student modeling being used?

# • Intelligent Tutoring Systems

**did I get this**

Here are the number of hours that 9 students spend on the computer on a typical day:

1 6 7 5 8 5 11 12 15

What is the median number of hours spent on the computer?

5  
  6.5  
  7  
  7.5  
  8

[Reset this Activity](#)

Image from the Statistics and Probability Course from the Open Learning Initiative

## Student modeling and its context

# • Intelligent Tutoring Systems

**did I get this**

Here are the number of hours that 9 students spend on the computer on a typical day:

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5  
  6.5  
  7  
  7.5  
  8

✖ Incorrect. This is the mode, or the most frequently occurring number. The median is the middle most number,  $(n + 1)/2$ , in an ordered list of values.

[Reset this Activity](#)

Image from the Statistics and Probability Course from the Open Learning Initiative

# Student modeling and its context

# • Intelligent Tutoring Systems

did I get this

Here are the number of hours that 9 students spend on the computer on a typical day:

1 6 7 5 8 5 11 12 15

What is the median number of hours spent on the computer?

5  
  6.5  
  7  
  7.5  
  8

Correct. After you order the data, since  $n = 9$ , the median is  $(9 + 1)/2 = 5$ , or the 5th observation in the ordered list, which in this case is 7.

[Reset this Activity](#)

Image from the Statistics and Probability Course from the Open Learning Initiative

# Student modeling and its context

Analyze real world problem scenarios

Tracked by knowledge tracing

23  
2-  
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SysFB09

Hint

Done

Skills

Scenario

My current cell phone company charges me \$14.95 per month for service and \$.13 per minute. PPS Cellular Phone Company has offered me \$15.00 worth of free calls a month if I switch, but the charge is \$.39 per minute.

1. How many minutes of calls can I get from PPS Cellular Phone Company for \$50? What is the cost from my current company for that number of minutes?
2. How many minutes of calls can I get from my current company for fifty dollars? What is the cost from PPS Cellular Phone Company for that number of minutes?
3. What is the cost from both companies for sixty minutes of calls?

Use equations, symbolic calculator

Clear All

$$0.13t + 14.95 - 0.13t = 0.39t - 15 - 0.13t$$

$$14.95 = 0.26t - 15$$

Add 15 to both sides

$$14.95 + 15 = 0.26t - 15 + 15$$

$$29.95 = 0.26t$$

Divide both sides by 0.26

Worksheet

Quantity Name	Time	Current cost	PPS cost
Unit	minutes	\$	\$
Expression	$t$	$0.13t + 14.95$	$0.39t - 15.00$
Question 1	166.6667	36.6167	
Question 2	269.6154	50.00	
Question 3	60	22.75	
Question 4	115.1923	29.925	

Use table, spreadsheet

Grapher

X Interval

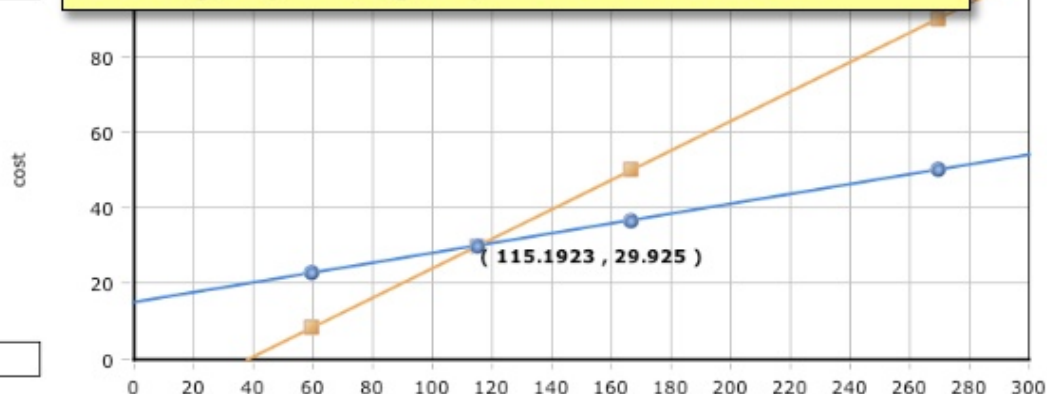
20

Y Interval

20

100

Use graphs, graphics calculator



0

0

Time ( minutes )

300

Legend:  Current cost

$y = 0.13t + 14.95$

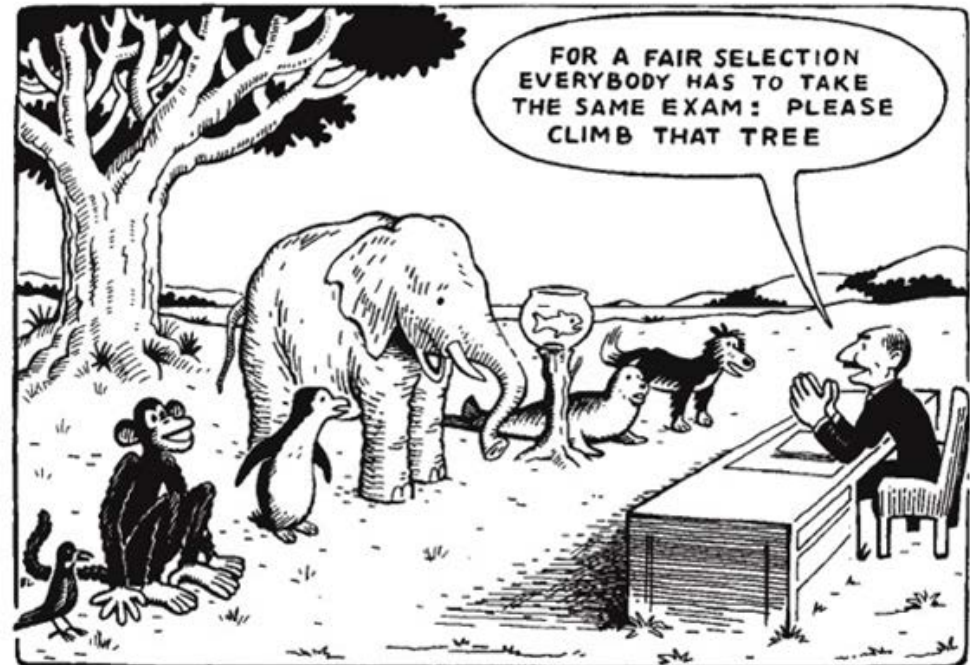
 PPS cost

$y = 0.39t - 15$

Model tracing to provide context-sensitive Instruction

## ITS detect and fit to individual differences in:

- student knowledge,
- engagement, and
- motivation,



# Student modeling and its context



The area of study covering the set of tools and techniques to achieve this assessment is student modeling

**Student modeling and its context**



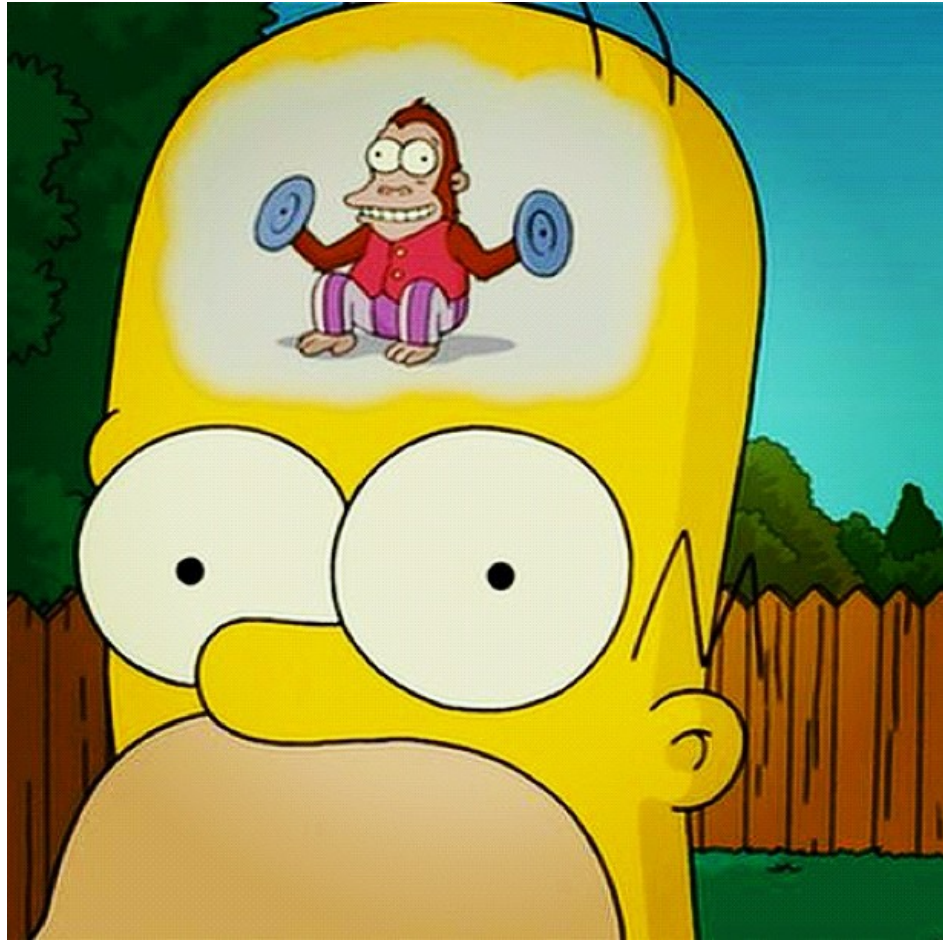


**NO  
 WHATSAPP  
 EXAM  
 TIME  
 NOW! >\_<**



# Student modeling and its context

... while student performance is observable, student knowledge remains latent.



**Student modeling and its context**



## Knowledge Component

A skill or knowledge component is a description of a mental structure or process that a learner uses, alone or in combination with other knowledge components, to accomplish steps in a task or a problem.

(Koedinger et al, 2012)

# Student modeling and its context

- **Student Models**

- Bayesian Knowledge Tracing (BKT)

- Corbett, Anderson, 1995

- Performance Factors Analysis (PFA)

- Pavlik, Cen, Koedinger, 2009

- ELO Rating System

- Pelánek, 2014

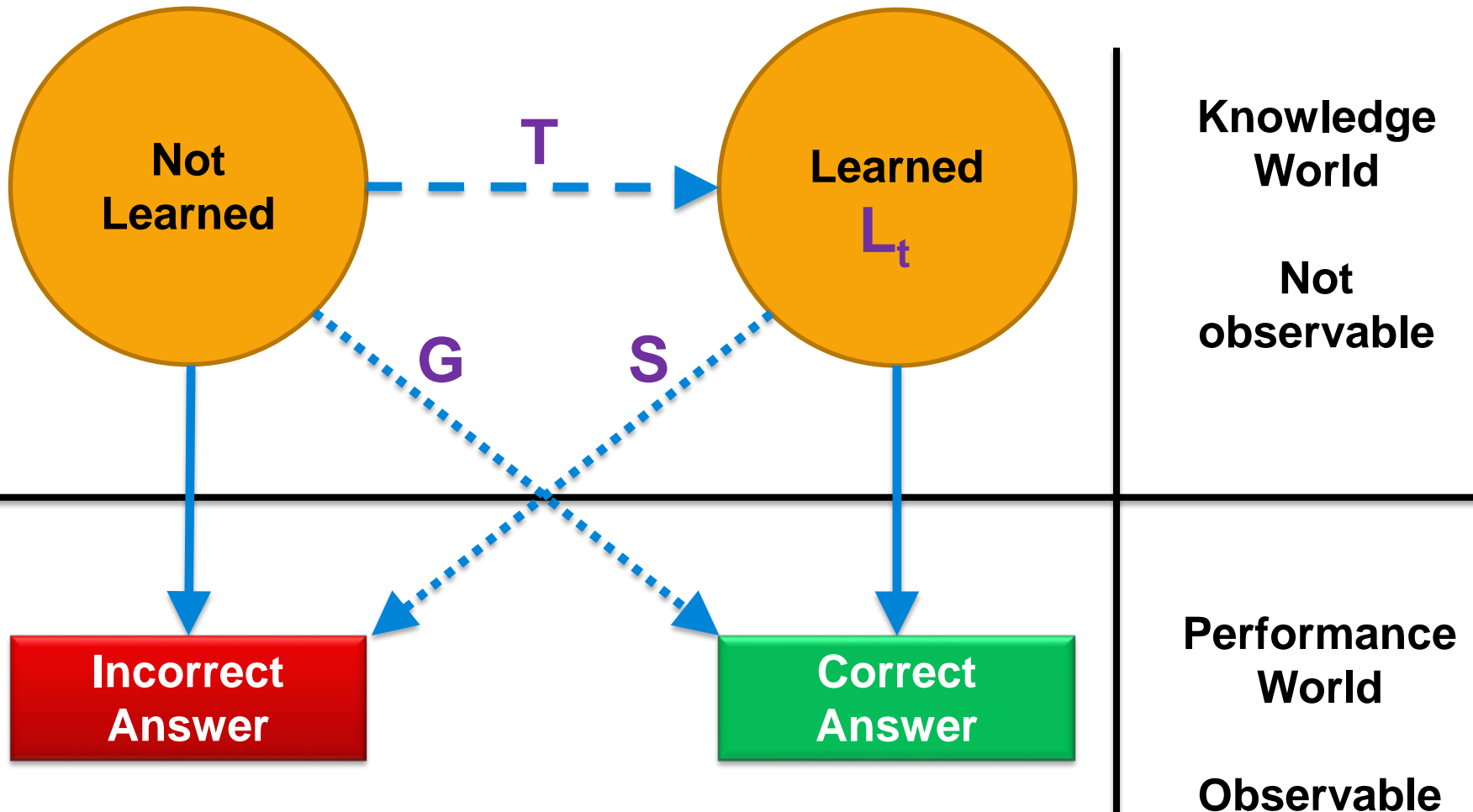
- Item Response Theory

- ...

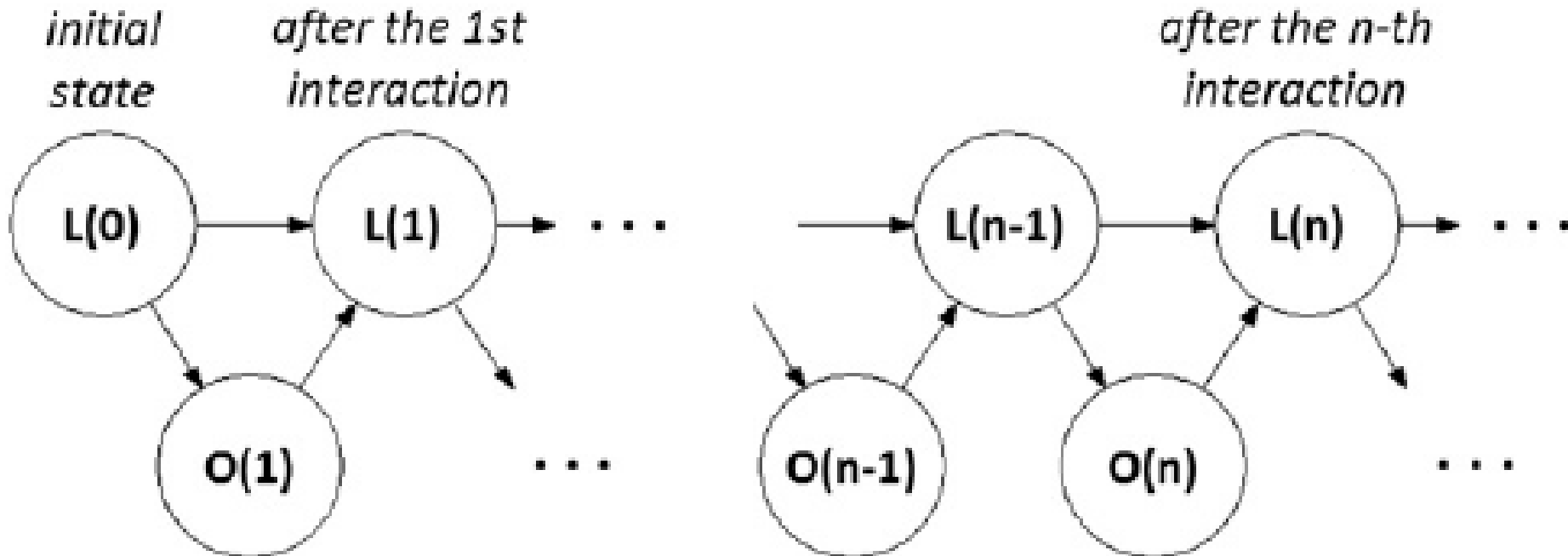
**Student modeling and its context**

# Bayesian Knowledge Tracing (BKT)

- BKT is a model used to infer student's knowledge given their history of responses to problems and can be used to predict future performance.
- BKT is a two state Hidden Markov Model, these states being the one in which the student knows a given skill, and the one where the student does not. Once the student knows a skill, it will not be forgotten
- Usually, a separate BKT model is fit for each skill and only the first attempt at each question is taken for each student.



- ... So in general ...



$$P(L_{t-1}|Correct_t) = \frac{P(L_{t-1}) \cdot (1 - S)}{P(L_{t-1}) \cdot (1 - S) + (1 - P(L_{t-1})) \cdot G} \quad \text{Eq. 1}$$

$$P(L_{t-1}|Incorrect_t) = \frac{P(L_{t-1}) \cdot S}{P(L_{t-1}) \cdot S + (1 - P(L_{t-1})) \cdot (1 - G)} \quad \text{Eq. 2}$$

$$P(L_t) = P(L_{t-1}|Action_t) + (1 - P(L_{t-1}|Action_t)) \cdot T \quad \text{Eq. 3}$$

$$C_{t+1} = P(L_t) \cdot (1 - S) + (1 - P(L_t)) \cdot G \quad \text{Eq. 4}$$

Where,

- $L_0$ , the probability a student knows the skill before attempting the first problem,
- $T$ , is the transition probability at each practice opportunity,
- $G$ , the probability of Guessing
- $S$ , the probability of Slipping
- $L_t$  is the probability of knowing a skill at the “time-attempt” point  $t$
- $C_{t+1}$  is the probability of answering correctly the next question



## Skill: Calculate the median

$$L_0 = 0.25$$

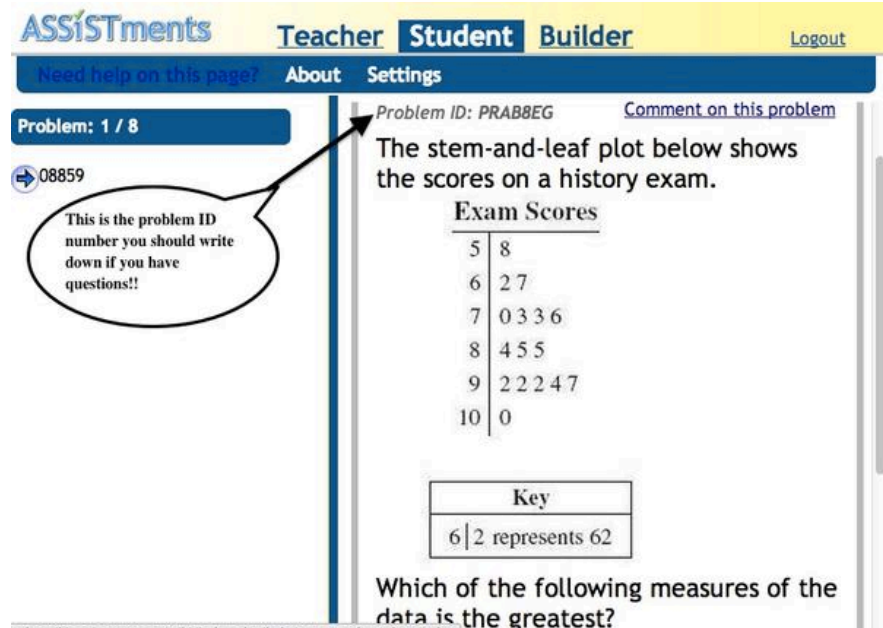
$$T = 0.2$$

$$S = 0.15$$

$$G = 0.1$$

Student	Right	$L_t$	$P(L_{t-1}   \text{Action}_t)$	$C_{t+1}$
Student 1	0	0,230	0,038	0,323
Student 1	1	0,714	0,642	0,685
Student 1	1	0,950	0,937	0,862
Student 1	0	0,752	0,690	0,714
Student 1	1	0,958	0,948	0,869

- ASSISTments:**  
 (<https://www.assistments.org/>) This intelligent tutor developed by the Worcester Polytechnic Institute is used by more than 600 teachers from 42 American states and 14 countries and their students solved  $10^6$  problems in 2015.



The screenshot shows the ASSISTments interface with the following elements:

- Navigation tabs: ASSISTments, Teacher, Student, Builder, Logout
- Sub-navigation: Need help on this page?, About, Settings
- Problem ID: 08859 (highlighted with a callout bubble: "This is the problem ID number you should write down if you have questions!!")
- Problem ID: PRABBEG
- Text: "The stem-and-leaf plot below shows the scores on a history exam."
- Stem-and-leaf plot titled "Exam Scores":
 

Stem	Leaf
5	8
6	2 7
7	0 3 3 6
8	4 5 5
9	2 2 2 4 7
10	0
- Key: 6 | 2 represents 62
- Question: "Which of the following measures of the data is the greatest?"

## Where is Student modeling being used?

**Algebra Cognitive Tutor**

Tracked by knowledge tracing

Analyze real world problem scenarios

Use equations, symbolic calculator

Model tracing to provide context-sensitive Instruction

Use table, spreadsheet

Use graphs, graphics calculator

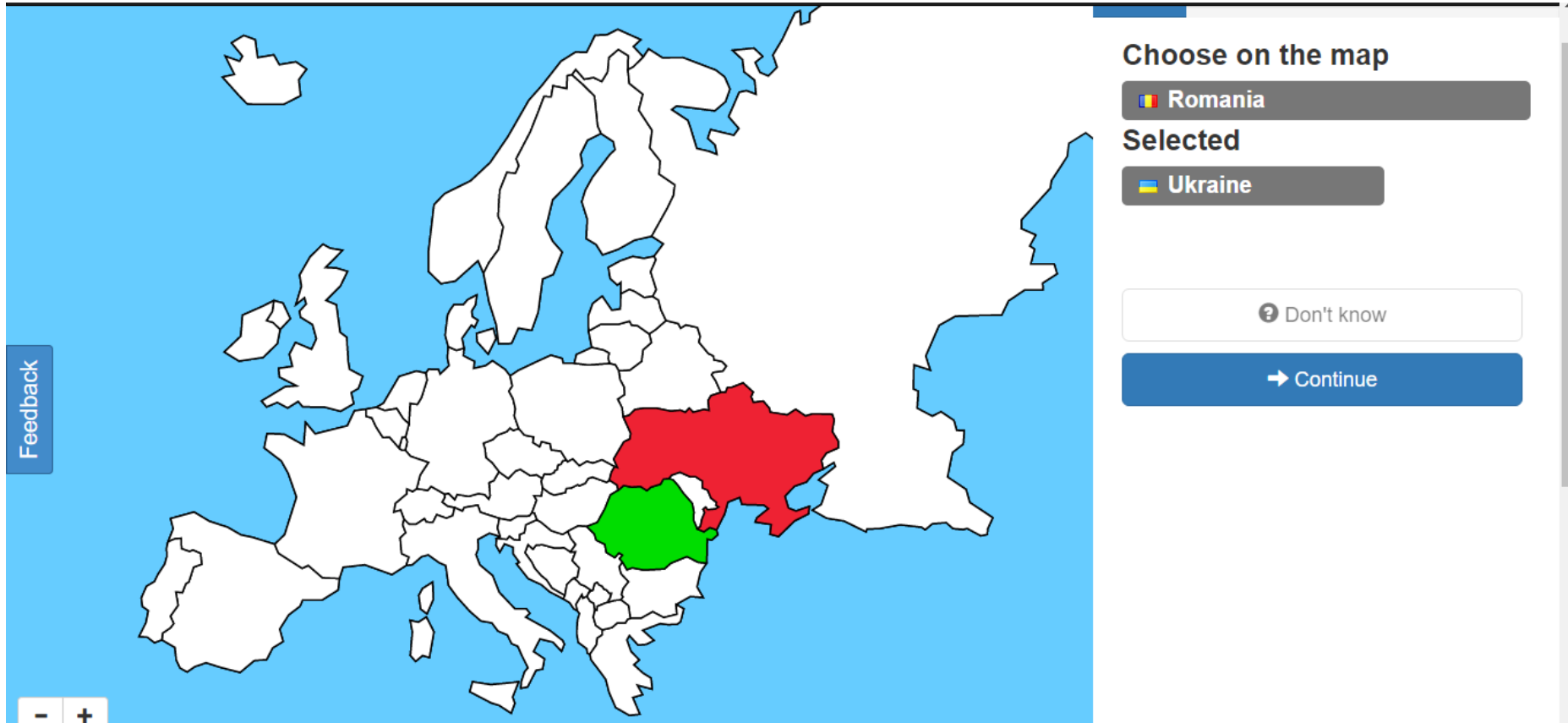
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Grapher

Legend:   
● Current cost  $y = 0.13t + 14.95$   
● PPS cost  $y = 0.39t - 15$

• **Cognitive tutors from Carnegie Learning Inc: Spin-off from Carnegie Mellon University**  
 Over 500 000 students per year  
 The Algebra Tutor is being used by more than 17 000 students in 147 schools

**Where is Student modeling being used?**



Feedback

Choose on the map

- Romania
- Ukraine

Selected

- Don't know

→ Continue

- +

Outline maps from Masaryk University Brno



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Research Group

# Thanks for your attention Questions?

# Thanks for your support



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